Advanced Algebra w/ Trig   Name:

6.6 Solving Radical Equations

Solve the following radical equations for x. Be sure to check for extraneous solutions!

1. $5^2 = (\sqrt{4-6x})^2$

$25 = 4 - 6x$

$\dfrac{21}{-6} = \dfrac{-6x}{-6}$

$\boxed{-\dfrac{21}{6} = x}$

Check:

$\sqrt{4 - 6(-\frac{21}{6})}$

$\sqrt{4 + 21}$

$\sqrt{25} = 5 \checkmark$

2. $(\sqrt[3]{2x})^3 = 4^3$

$2x = 64$

$\boxed{x = 32}$

Check:

$\sqrt[3]{2(32)} = \sqrt[3]{64}$

$= 4$

3. $(\sqrt{x-5})^2 = 4^2$

$x - 5 = 16$

$\boxed{x = 21}$

Check:

$\sqrt{21 - 5}$

$\sqrt{16} = 4 \checkmark$

4. $(\sqrt{3x+1})^2 = (-5)^2$

$3x + 1 = 25$

$3x = 24$

$\cancel{x = 8}$ extraneous!

$\boxed{\text{no real solution}}$

Check:

$\sqrt{3(8)+1}$

$\sqrt{24+1}$

$\sqrt{25} = 5$ & $5 \neq -5$

5. $(\sqrt[3]{3x-6})^3 = 3^3$

$3x - 6 = 27$

$3x = 33$

$\boxed{x = 11}$

Check:

$\sqrt[3]{3(11)-6}$

$\sqrt[3]{33-6}$

$\sqrt[3]{27} = 3 \checkmark$

6. $2 + \sqrt{3x+7} = 6$

$\dfrac{-2 \qquad\qquad -2}{}$

$(\sqrt{3x+7})^2 = 4^2$

$3x + 7 = 16$

$3x = 9$

$\boxed{x = 3}$

Check:

$2 + \sqrt{3(3)+7}$

$2 + \sqrt{16}$

$2 + 4 = 6 \checkmark$

7. $\sqrt{x-4} - 1 = 5$

$\dfrac{+1 \quad +1}{}$

$(\sqrt{x-4})^2 = 6^2$

$x - 4 = 36$

$\boxed{x = 40}$

Check:

$\sqrt{40-4} - 1$

$\sqrt{36} - 1$

8. $(2x+3)^{\frac{1}{3}} = 2$

$(\sqrt[3]{2x+3})^3 = 2^3$

$2x + 3 = 8$

$2x = 5$

$\boxed{x = \dfrac{5}{2}}$

Check:

$(2(\frac{5}{2})+3)^{1/3}$

$(5+3)^{1/3}$

$8^{1/3} = \sqrt[3]{8} = 2 \checkmark$

$x - 4 = 36$
$\boxed{x = 40}$

$\sqrt{40 - 4} \; -1$
$\sqrt{36} \; -1$
$6 - 1 = 5 \checkmark$

$\boxed{x = \dfrac{5}{2}}$

$8^{1/3} = \sqrt[3]{8} = 2\checkmark$

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9. $\left((2x-4)^{\frac{1}{4}}\right)^4 = (-2)^4$

$2x - 4 = 16$
$2x = 20$
$\cancel{x = 10}$
extraneous

$\boxed{\text{no real solution}}$

check:
$(2(10)-4)^{1/4}$
$(16)^{1/4}$
$\sqrt[4]{16} = 2$
&
$2 \neq -2$

10. $4 - (1-7x)^{\frac{1}{3}} = 0$
$\underline{-4 \qquad\qquad -4}$
$\dfrac{-(1-7x)^{1/3}}{-1} = \dfrac{-4}{-1}$
$\left[(1-7x)^{1/3}\right]^3 = 4^3$
$1 - 7x = 64$
$-7x = 63$
$\boxed{x = -9}$

check:
$4 - (1-7(-9))^{1/3}$
$4 - (1+63)^{1/3}$
$4 - \sqrt[3]{64}$
$4 - 4 = 0 \checkmark$

11. $\left(\sqrt{3x-2}\right)^2 = \left(\sqrt{x-4}\right)^2$

$3x - 2 = x - 4$
$2x = -2$
$\cancel{x = -1}$

No real solutions

extraneous (b/c can't $\sqrt{}$ a neg. #)

Check
$\sqrt{3(-1)-2} = \sqrt{-1-4}$
$\sqrt{-5} \cancel{=} \sqrt{-5}$

12. $\left(\sqrt{x+1}\right)^2 = \left(\sqrt{2x-7}\right)^2$
$x + 1 = 2x - 7$
$\boxed{8 = x}$

Check:
$\sqrt{8+1} \overset{?}{=} \sqrt{2(8)-7}$
$\sqrt{9} \overset{?}{=} \sqrt{9}$
$\checkmark$

13. $\left(\sqrt{x-1}\right)^2 = \left(4\sqrt{x+1}\right)^2$

$x - 1 = 4^2(x+1)$
$x - 1 = 16(x+1)$
$x - 1 = 16x + 16$
$-17 = 15x$
$x = -\dfrac{17}{15}$

14. $\left(\sqrt{3x+2}\right)^2 = \left(5\sqrt{x-7}\right)^2$

$3x + 2 = 25(x-7)$
$3x + 2 = 25x - 175$
$\underline{-3x \qquad\quad -3x}$
$2 = 22x - 175$
$\underline{+175 \qquad\quad +175}$
$177 = 22x$

$-11-10x$

$$x = -\frac{17}{15}$$

Check:

$$\sqrt{-\frac{17}{15} - 1} = 4\sqrt{-\frac{17}{15} + 1}$$

$$\sqrt{-\frac{32}{15}} = 4\sqrt{-\frac{2}{15}}$$

✗ can't do $\sqrt{}$ of a neg. #

$$\frac{177}{22} = \frac{22x}{22}$$

$$\boxed{\frac{177}{22} = x}$$