1. Simplify

   a. $\dfrac{x^7}{x^3} = X^{7-3} = \boxed{X^4}$

   b. $\left(x^3\right)^6 = X^{3 \cdot 6} = \boxed{X^{18}}$

   c. $\sqrt[4]{14e^{12}} = \boxed{e^3 \cdot \sqrt[4]{14}}$

   d. $\sqrt{49e^6} \cdot 2e^{-3}$ $7e^3 \cdot 2e^{-3} = 14e^0 = \boxed{14}$

   e. $\dfrac{e^{x-4}}{e^{3x}}$ $e^{X-4-3x} = \boxed{e^{-2x-4}}$

   f. $\log_5 25^{(x+1)}$ $5^? = 25^{x+1}$
   $5^{\textcircled{?}} = (5^{\textcircled{2}})^{x+1}$
   $? = 2(x+1)$
   $= \boxed{2x+2}$

2. Let $f(x) = 3x^2 - 5$ and $g(x) = 5x^2$. Find:

   a. f(x)• g(x)
   b. f(x)+g(x)
   c. g(f(x))
   d. f(g(x))

   a) $(3x^2 - 5)(5x^2) = \boxed{15x^4 - 25x^2}$

   b) $3x^2 - 5 + 5x^2 = \boxed{8x^2 - 5}$

   c) $5(3x^2 - 5)^2 = 5(3x^2 - 5)(3x^2 - 5)$
   $= 5(9x^4 - 30x^2 + 25)$
   $= \boxed{45x^4 - 150x^2 + 125}$

   d) $3(5x^2)^2 - 5 = 3(25x^4) - 5$
   $= \boxed{75x^4 - 5}$

3. Find the inverse for the following:

   a. $f(x) = x^2 + 4$
   b. $f(x) = \sqrt{(4x-2)} + 3$
   c. $y = \log_4 x + 4$

   a) $x = y^2 + 4$
   $X - 4 = y^2$ SO $\boxed{f^{-1}(x) = \sqrt{x-4}}$

   b) $x = \sqrt{(4y-2)} + 3$
   $-3 \qquad\qquad -3$
   $X - 3 = \sqrt{4y-2}$
   $(x-3)^2 = 4y-2$
   $X^2 - 6x + 9 = 4y - 2$
   $\dfrac{X^2 - 6x + 11}{4} = y$ SO $\boxed{f^{-1}(x) = \dfrac{X^2 - 6x + 11}{4}}$

   c) $x = \log_4 y + 4$
   $X - 4 = \log_4 y$
   $4^{x-4} = y$ SO $\boxed{y = 4^{x-4}}$

4. State the range and domain for

   a. $y = \dfrac{1}{4}\sqrt[3]{x+4} - 2$ y's * by $\frac{1}{4}$
   $\leftarrow 4$
   $\downarrow 2$

   b. $y = \sqrt{x+1} + 14$ $\leftarrow 1, \uparrow 14$

   c. $f(x) = 2 \cdot 4^x + 3$ exp. growth, $\uparrow 3$

   d. $g(x) = \log_4(x-1) + 3$ $\log_4(x)$
   $\rightarrow 1$
   $\uparrow 3$

   a) D: $[-4, \infty)$ R: $[-2, \infty)$
   b) D: $[1, \infty)$ R: $[14, \infty)$
   c) D: $(-\infty, \infty)$ R: $[3, \infty)$
   d) D: $(1, \infty)$ R: $(-\infty, \infty)$

5. If the graph $y = \sqrt[3]{x}$ is shifted 3 units up and 2 units to the right, what is the equation of the translated graph?

   $\boxed{y = \sqrt[3]{x-2} + 3}$

6. Solve

a. $\sqrt{3x} - 81 = 0$
b. $\sqrt{x+4} = x+2$
c. $\sqrt{x+5} = \sqrt{x-2} + 3$
d. $\log x + \log (2x+30) = 3$
e. $\log_3 x = \log_3 (2x-5)$

be sure to check for extraneous solutions

a) $(\sqrt{3x})^2 = (81)^2$
$3x = 6561$
$\boxed{x = 2187}$

check: $\sqrt{3 \cdot 2187} - 81 = 0$
$81 - 81 = 0$
✓

b) $(\sqrt{x+4})^2 = (x+2)^2$
$x+4 = (x+2)(x+2)$
$x+4 = x^2 + 4x + 4$
$0 = x^2 + 3x$
$0 = x(x+3)$
$\boxed{0 = x}$ & $0 = x+3$
$\cancel{-3=x}$
extraneous

check: $\sqrt{0+4} = 0+2$
$\sqrt{4} = 2$
$2 = 2$ ✓

$\sqrt{-3+4} = -3+2$
$\sqrt{1} = -1$
$1 \neq -1$ ✗

c) $(\sqrt{x+5})^2 = (\sqrt{x-2} + 3)^2$
$x+5 = (\sqrt{x-2} + 3)(\sqrt{x-2} + 3)$
$x+5 = x-2 + 6\sqrt{x-2} + 9$
$-2 = 6\sqrt{x-2}$

$(-\frac{1}{3})^2 = (\sqrt{x-2})^2$
$\frac{1}{9} = x-2$
$\boxed{\frac{19}{9} = x}$

d) $\log x + \log(2x+30) = 3$
$\log(x[2x+30]) = 3$
$\log_{10}(2x^2 + 30x) = 3$
$10^3 = 2x^2 + 30x$
$0 = 2x^2 + 30x - 1000$
$0 = 2(x^2 + 15x - 500)$

so $\boxed{x = \dfrac{-15 + \sqrt{2225}}{2}}$

need to use quad. formula
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \dfrac{-15 \pm \sqrt{15^2 - 4(1)(-500)}}{2(1)}$
$x = \dfrac{-15 \pm \sqrt{2225}}{2}$
neg. doesn't work → extraneous (can't do log of a neg. #)

e) $\log_3 x = \log_3 (2x-5)$
$x = 2x-5$
$-x = -5$
$\boxed{x=5}$

7. Gasoline currently costs $3.99 per gallon. If the price per gallon increases an average of 6% every month, what function models the price after x months?

$y = 3.99(1+0.06)^x$
$\boxed{y = 3.99(1.06)^x}$

8. Write a function that
   a. represents exponential decay
   b. represents exponential growth
   $y = a \cdot b^x$

a) any function where $b > 1$
EX: $y = \frac{1}{2} \cdot 2^x$

b) any function where $b < 1$
EX: $y = 2 \cdot (\frac{3}{4})^x$

9. You deposit $6000 in an account that pays 10% annual interest, compounded continuously. Approximately how long will it take for the balance

10. Rewrite $\log_4 8 = \frac{3}{2}$ in exponential form.

annual interest, compounded continuously.
Approximately how long will it take for the balance
to double? $A = Pe^{rt}$ $6000 \times 2 = 12{,}000$

$$\frac{12000}{6000} = \frac{6000 e^{0.1t}}{6000}$$

$$2 = e^{0.1t}$$

$0.1t = \log_e 2$
$0.1t = \ln 2$
$0.1t = .693$
$t = \boxed{6.93 \text{ years}}$

12. Expand $\ln \dfrac{3t^4}{7v^2}$

$\ln 3t^4 - \ln 7v^2$
$(\ln 3 + \ln t^4) - (\ln 7 + \ln v^2)$
$(\ln 3 + 4\ln t) - (\ln 7 + 2\ln v)$
$\boxed{\ln 3 + 4\ln t - \ln 7 - 2\ln v}$

10. Rewrite $\log_4 8 = \dfrac{3}{2}$ in exponential form.

$$\boxed{4^{\frac{3}{2}} = 8}$$

11. Condense $\log 7 + \log 2 - \log 10 + \log 5$

$\log(7 \cdot 2) - \log 10 + \log 5$
$\log 14 - \log 10 + \log 5$
$\log\left(\frac{14}{10}\right) + \log 5$
$\log\left(\frac{7}{5}\right) + \log 5$
$\log\left(\frac{7}{5} \cdot 5\right)$
$\boxed{\log 7}$