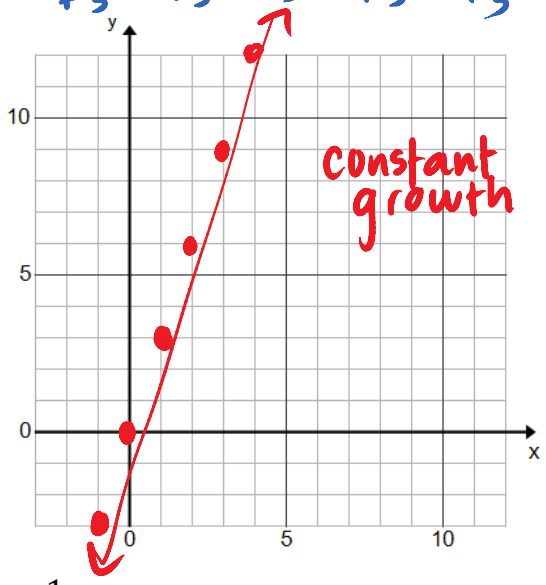
8.5 & 8.6 Exponential Functions

1. $y = 3x$ (Linear Model)

x	-1	0	1	2	3	4
y	-3	0	3	6	9	12

+1 +1 +1 +1 +1

+3 +3 +3 +3 +3

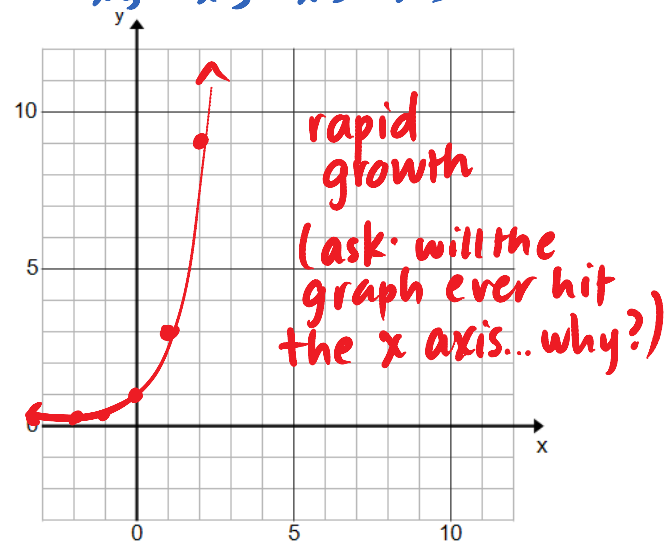


2. $y = 3^x$ (Exponential Model)

x	-2	-1	0	1	2	3
y	1/9	1/3	1	3	9	27

+1 +1 +1 +1 +1

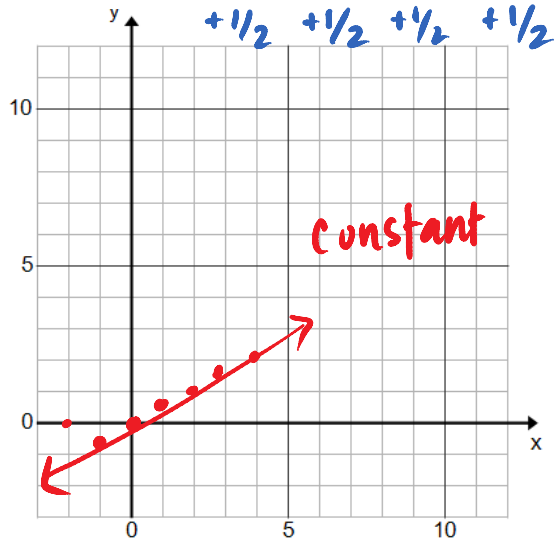
×3 ×3 ×3 ×3 ×3



3. $y = \frac{1}{2}x$

x	-1	0	1	2	3	4
y	-1/2	0	1/2	1	3/4	2

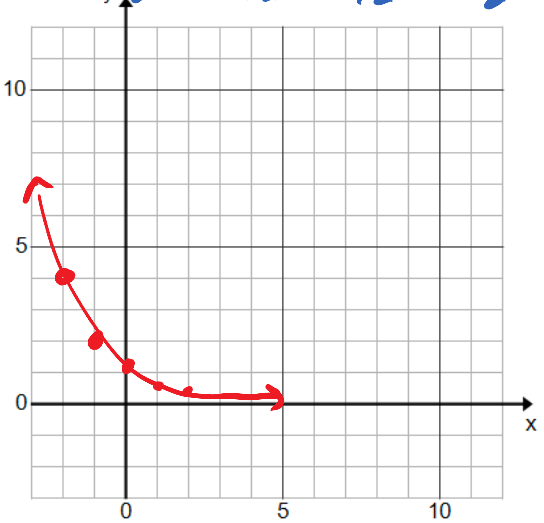
+1/2 +1/2 +1/2 +1/2



4. $y = \left(\frac{1}{2}\right)^x$

x	-2	-1	0	1	2	3
y	4	2	1	1/2	1/4	1/8

·1/2 ×1/2 ×1/2 ×1/2 ×1/2



Compare the two graphs: How are the similar? How are they different? (Hint: How does each graph "grow")

Constant growth vs. rapid/exponential growth OR decay

The BASIC form of an exponential function is $y = (b)^x$

Think about

$y = 2x$

$y = 2(1) = 2$
$y = 2(2) = 4$
$y = 2(3) = 6$
$y = 2(4) = 8$

$y = 2^x$

$y = 2^1 = 2$
$y = 2^2 = 2 \cdot 2 = 4$
$y = 2^3 = 2 \cdot 2 \cdot 2 = 8$
$y = 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

$y = \left(\frac{1}{2}\right)^x$

$y = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$
$y = \left(\frac{1}{2}\right)^2 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$
$y = \left(\frac{1}{2}\right)^3 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$
$y = \left(\frac{1}{2}\right)^4 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$

BIG IDEA: $y = b^x$

If b > 1, then it is exponential growth,

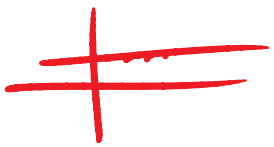
If 0 < b < 1, then it is exponential decay.

Think about it: What would an exponential function look like if b is one? Make a table to help you answer.

X	0	1	2	3	4	5
y	1	1	1	1	1	1

$y = (1)^x$

not growth
constant
horizontal
line

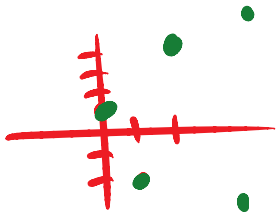


Challenge: What would an exponential function look like if b is negative? Make a table to help you answer.

X	0	1	2	3	4	5
y	1	-2	4	-8	16	-32

$y = (-2)^x$

not exp. growth



?!

Determine if each of the following represents GROWTH or DECAY

1) $y = 5^x$

5.5.5... growth

2) $y = \left(\frac{1}{2}\right)^x$

$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$ ... decay

3) $y = (3)^x + 6$

growth

4) $y = \left(\frac{1}{4}\right)^{x+3} + 1$

decay

5) $y = \left(\frac{3}{2}\right)^x$

growth

6) $y = \left(\frac{1}{3}\right)^{-x}$ $y = \left(\frac{3}{1}\right)^x$

growth

7) $y = (5)^{-x} + 3$

decay

8) $y = (.99)^x - 3$

decay

Linear or exponential? Decide if the equation is linear or exponential and then write an equation that represents the "rule" for the table:

Exponential Growth

x	-2	-1	0	1	2	3
y	2	4	8	16	32	64

+1 +1 +1 +1 +1

×2 ×2 ×2 ×2 ×2

$y = 8 \cdot (2)^x$

check...

$y = 8(2)^1 = 16$

$y = 8(2)^2 = 32$

Exponential Growth

x	-2	-1	0	1	2	3
y	$\frac{2}{9}$	$\frac{2}{3}$	2	6	18	54

+1 +1 +1 +1 +1

×3 ×3 ×3 ×3 ×3

$y = 2 \cdot (3)^x$

check

$y = 2(3)^1 = 6$

$y = 2(3)^2 = 18$

x	-2	-1	0	1	2	3
y	-4	-1	2	5	8	11

+1 +1 +1 +1 +1

+3 +3 +3 +3 +3

constant...(linear) growth

$m = \frac{\Delta y}{\Delta x} = \frac{3}{1} = 3$

$y = 3x + 2$

$b = (0, 2)$

x	-2	-1	0	1	2	3
y	32	16	8	4	2	1

+1 +1 +1 +1 +1

×1/2 ×1/2 ×1/2 ×1/2

exponential decay

$y = 8\left(\frac{1}{2}\right)^x$

x	-2	-1	0	1	2	3
y	243	81	27	9	3	1

×1/3 ×1/3 ×1/3 ×1/3

exponential decay

$y = 27 \cdot \left(\frac{1}{3}\right)^x$

x	-2	-1	0	1	2	3
y	16	13	10	7	4	1

+1 +1 +1 +1 +1

-3 -3 -3 -3 -3

constantly decreasing (linear)

$m = \frac{\Delta y}{\Delta x} = \frac{-3}{1}$

$b = (0, 10)$

$y = -3x + 10$