

# Law of Cosines

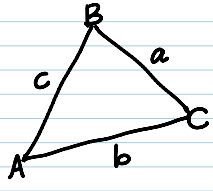
- works with cases of SSS & SAS

Law of cosines

If  $\triangle ABC$  has side lengths of  $a, b, c$  then:

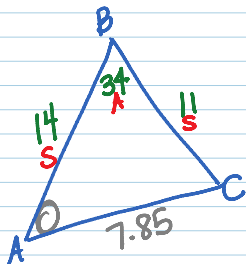
$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$


## Examples

- ① Solve  $\triangle ABC$  with  $a=11, c=14$ , &  $B=34^\circ$ .



SAS

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$b^2 = (11)^2 + (14)^2 - 2(11)(14) \cos 34$$

$$\sqrt{b^2} = \sqrt{61.66}$$

$$b = 7.85$$

~~$$\frac{\sin A}{11} = \frac{\sin 34}{7.85}$$~~

$$C = 180 - (51.59 + 34)$$

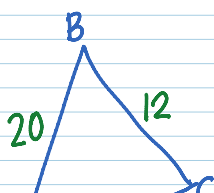
$$C = 94.4^\circ$$

$$\frac{7.85 \sin A}{7.85} = \frac{11 \sin 34}{7.85}$$

$$\sin A = .78$$

$$A = \sin^{-1}(.78) = 51.59^\circ$$

- ② Solve  $\triangle ABC$  with  $a=12, b=27, c=20$ .



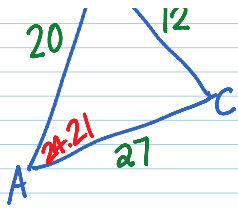
SSS

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$12^2 = 27^2 + 20^2 - 2(27)(20) \cos A$$

$$144 = 1129 - 1080 \cdot \cos A$$

$$-1129 \quad -1129$$



$$\frac{\sin C}{20} = \frac{\sin 24.21}{12}$$

$$\sin C = \frac{20 \sin 24.21}{12}$$

$$\sin C = .68$$

$$C = \sin^{-1}(.68) = 43.12^\circ$$

$$144 = 1129 - 1080 \cos A$$

$$\frac{-985}{-1080} = \frac{-1080 \cos A}{-1080}$$

$$.91 = \cos A$$

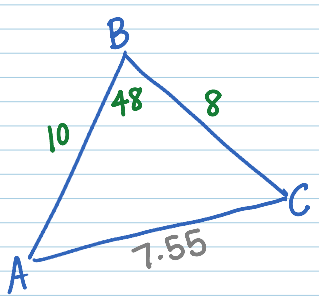
$$\cos^{-1}(.91) = A$$

$$24.21^\circ = A$$

$$B = 180 - (43.12 + 24.21)$$

$$B = 112.67^\circ$$

③ Solve  $\triangle ABC$  with  $a=8, c=10, B=48^\circ$ .



$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$b^2 = (8)^2 + (10)^2 - 2(8 \times 10) \cos 48$$

$$b^2 = 56.94$$

$$b = 7.55$$

$$\angle C$$

$$180 - (48 + 51.95)$$

$$80.05^\circ$$

$$\angle A$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{8} = \frac{\sin 48}{7.55}$$

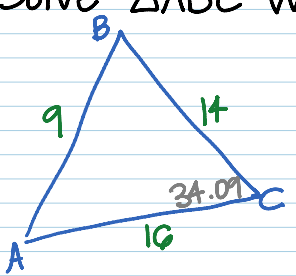
$$\frac{7.55 \sin A}{7.55} = \frac{8 \sin 48}{7.55}$$

$$\sin A = .79$$

$$A = \sin^{-1}(.79)$$

$$A = 51.95^\circ$$

④ Solve  $\triangle ABC$  with  $a=14, b=16, c=9$ .



$$c^2 = a^2 + b^2 - 2abc \cos C$$

$$9^2 = (14)^2 + (16)^2 - 2(14 \times 16) \cos C$$

$$81 = 452 - 448 \cos C$$

$$\frac{452 - 452}{-448} = \frac{-448 \cos C}{-448}$$

$$.83 = \cos C$$

$$\cos^{-1}(.83) = C$$

$$34.09^\circ = C$$

$$\frac{\sin A}{14} = \frac{\sin 34.09}{9}$$

$$\sin A = \frac{14 \sin 34.09}{9}$$

$$\sin A = .87$$

$$A = \sin^{-1}(.87)$$

$$A = 60.69^\circ$$

$$B = 180 - (34.09 + 60.69)$$

$$B = 85.22^\circ$$