Law of Sines Day 2: The Ambiguous Case

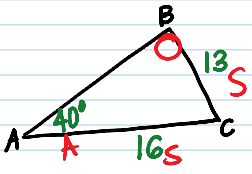
2 angles & side determine 1 triangle. (ASA, AAS)

BUT

2 sides & 1 angle (SSA) may determine 0, 1, or 2 triangles.

Examples: State the case (AAS, ASA or SSA). Then determine the # of triangles

① $A = 40°, a = 13, b = 16$



$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 40}{13} = \frac{\sin B}{16}$

$\frac{13 \sin B}{13} = \frac{16 \sin 40}{13}$

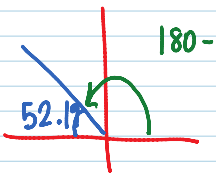
$\sin B = \frac{16 \sin 40}{13}$

$\sin^{-1}(\sin B) = \sin^{-1}(.79)$

$B = \sin^{-1}(.79)$

$B = 52.19$

$40 + 52.19 = 92.19$

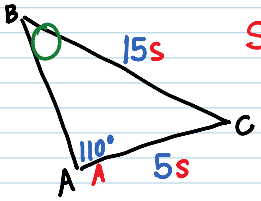


$180 - 52.19 = 127.81$

$127.81 + 40 = 167.81$

SSA
2 TRIANGLES

② $A = 110°, a = 15, b = 5$



SSA

SSA
1 triangle

$\frac{\sin B}{b} = \frac{\sin A}{a}$

$\frac{\sin B}{5} = \frac{\sin 110}{15}$

$\frac{15 \sin B}{15} = \frac{5 \sin 110}{15}$

$\sin B = \frac{5 \sin 110}{15}$

$\sin B = .31$

$B = \sin^{-1}(.31)$

$B = 18.06°$

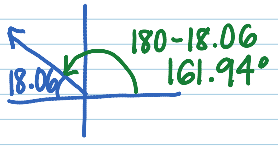
1 TRIANGLE?

$18.06 + 110 = 128.06$

less than 180

atleast 1 Δ

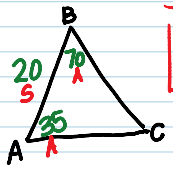
2 TRIANGLES?



$161.94 + 110 > 180$

no 2nd triangle

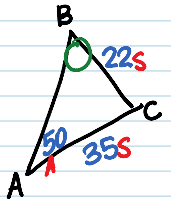
③ $A = 35°, B = 70°, c = 20$



ASA

1 triangle

④ A = 50°, a = 22, b = 35



SSA

$\frac{\sin B}{35} = \frac{\sin 50}{22}$

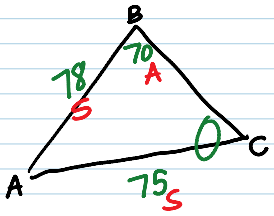
$\frac{22\sin B}{22} = \frac{35\sin 50}{22}$

$\sin B = 1.22$

$B = \sin^{-1}(1.22)$

NO TRIANGLE!

⑤ B = 70°, b = 75, c = 78



$\frac{\sin C}{78} = \frac{\sin 70}{75}$

$\sin C = \frac{78\sin 70}{75}$

$\sin C = .98$

$C = \sin^{-1}.98 = 77.76°$

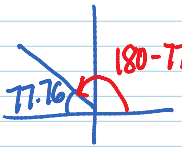
SSA
2 triangles

1 triangle?

$77.76 + 70 = 147.76$

↑ one triangle! less than 180

2 triangles?



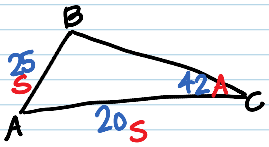
$102.24 + 70$

$172.24 < 180$

2 triangles

↑ less than 180!

⑥ C = 42°, c = 25, b = 20



$\frac{\sin B}{20} = \frac{\sin 42}{25}$

$\sin B = \frac{20\sin 42}{25}$

$B = \sin^{-1}(.53)$

$B = 32.36°$

SSA
one triangle

1 triangle? ✓

$32.36 + 42 < 180$ ✓

2 triangles? ✗

$147.63 + 42 > 180$ ✗

THE STEPS

① Find the ∠ opposite your given side

② Add it to the other given ∠. If the sum ≥ 180, NO △

If not, you have at least 1 triangle.

③ Check to see if there are 2 triangles. Find the $\not\angle$ in quad 2.

- add the $\not\angle$ to 180. If the sum ≥ 180, only 1 triangle. If not, you have 2 triangles!