

Warm-Up

Evaluate without a calculator:

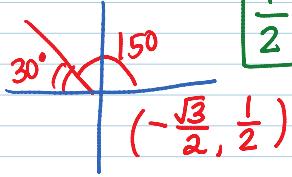
①  $\cos 45^\circ$

$$\frac{\sqrt{2}}{2}$$

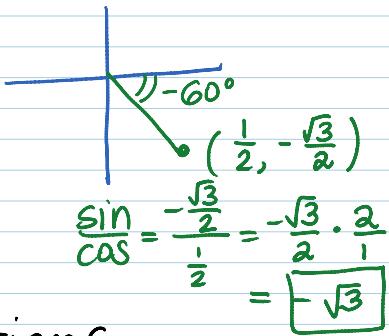
②  $\sin \frac{5\pi}{6}$

$$\frac{5\pi}{6} \times \frac{180}{\pi} = 150^\circ$$

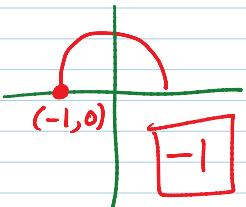
$$\boxed{\frac{1}{2}}$$



③  $\tan(-60^\circ)$



④  $\cos \pi$

13.4: Evaluate Inverse Trig Functions

In 13.3, we were given an angle & asked to evaluate the value of a trig function. Now in 13.4, we'll be given the value of a trig function & will be asked to find the angle.

So for example, find an angle whose  $\sin = \frac{1}{2}$ .

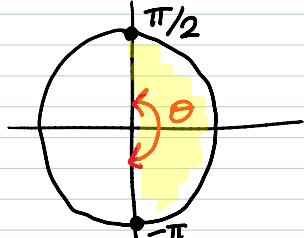
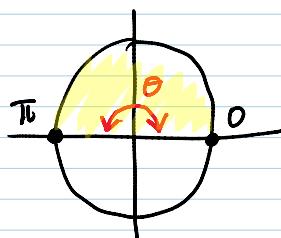
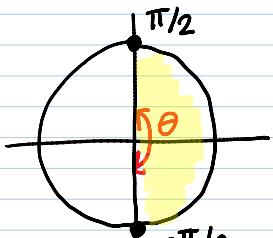
$$30^\circ, 150^\circ, 390^\circ, -330^\circ$$

To make things easier, we restrict the domain so we only have one unique angle.

Inverse Sine

Inverse Cosine

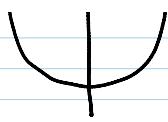
Inverse tangent





$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$-90^\circ < \theta < 90^\circ$$



$$0 < \theta < \pi$$

$$0 < \theta < 180^\circ$$



$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$-90^\circ < \theta < 90^\circ$$

Evaluate the expression in both radians & degrees.

a)  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$30^\circ \text{ or } \frac{\pi}{6}$$

b)  $\sin^{-1}(2)$

$$\sin\theta = 2$$

undefined

c)  $\tan^{-1}(-\sqrt{3})$

$$\tan\theta = -\sqrt{3}$$

$$-60^\circ \text{ or } -\frac{\pi}{3}$$

d)  $\sin^{-1}(-1)$

$$\sin\theta = -1$$

$$-90^\circ \text{ or } -\frac{\pi}{2}$$

e)  $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$$\cos\theta = -\frac{\sqrt{2}}{2}$$

$$\theta = 135^\circ \text{ or } \frac{3\pi}{4}$$

f)  $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$$\tan\theta = \frac{\sqrt{3}}{3}$$

$$\theta = 30^\circ \text{ or } \frac{\pi}{6}$$

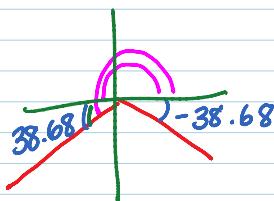
Take it a step further! Now solve over a given interval.

① Solve  $\sin\theta = -\frac{5}{8}$  where  $180^\circ < \theta < 270^\circ$

Step 1: use your calculator to find the x

$$\theta = 38.68^\circ$$

Step 2: find the indicated quadrant



$$180 + 38.68 = \boxed{218.68^\circ}$$

CHECK:  $\sin(218.68) = -\frac{5}{8}$

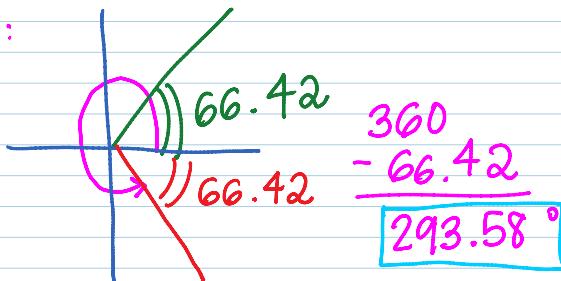
Solve for  $\theta$  over the given interval:

$$\textcircled{2} \cos \theta = 0.4; 270^\circ < \theta < 360^\circ$$

4<sup>th</sup> Quad

Step 1:  $\cos^{-1}(0.4) = 66.42$

Step 2:



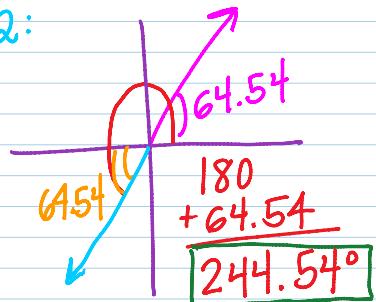
CHECK:  $\cos(293.53) = .4 \checkmark$

$$\textcircled{3} \tan \theta = 2.1, 180^\circ < \theta < 270^\circ$$

3<sup>rd</sup> Quad

Step 1:  $\tan^{-1}(2.1) \approx 64.54$

Step 2:



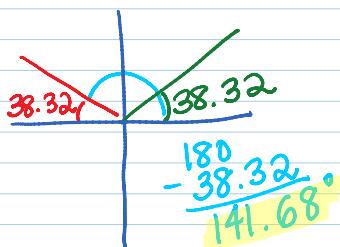
CHECK:  $\tan(244.54) = 2.1$

$$\textcircled{4} \sin \theta = 0.62; 90^\circ < \theta < 180^\circ$$

2<sup>nd</sup> Quad

Step 1:  $\sin^{-1}(0.62) \approx 38.32$

Step 2:



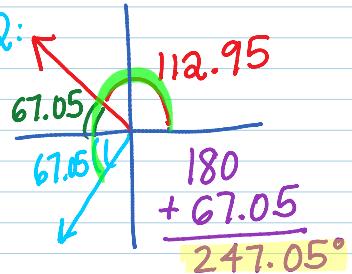
CHECK:  $\sin(141.68) = .62 \checkmark$

$$\textcircled{5} \cos \theta = -0.39, 180^\circ < \theta < 270^\circ$$

3<sup>rd</sup> Quad

Step 1:  $\cos^{-1}(-0.39) \approx 112.94$

Step 2:



CHECK:  $\cos(247.05) = -.39 \checkmark$